Quantitative Methods

Study Sessions 2 & 3

Weight on Exam	12%
SchweserNotes™ Reference	Book 1, Pages 95–346

Study Session 2: Quantitative Methods—Basic Concepts

The Time Value of Money

Cross-Reference to CFA Institute Assigned Reading #5

Time value of money (TVM) computations will be on your exam and understanding the material is essential for success not only for quantitative methods, but also other sections of the Level 1 exam. TVM is actually a larger portion of the exam than simply quantitative methods because of its integration with other topics. For example, any portion of the exam that requires discounting cash flows will require TVM calculations. This includes evaluating capital projects, using dividend discount models for stock valuation, valuing bonds, and valuing real estate investments. No matter where TVM shows up on the exam, the key to any TVM problem is to draw a timeline and be certain of when the cash flows will occur so you can discount those cash flows appropriately.

An interest rate can be interpreted as a required rate of return, a discount rate, or as an opportunity cost; but it is essentially the price (time value) of money for one period. When viewed as a required (equilibrium) rate of return on an investment, a nominal interest rate consists of a real risk-free rate, a premium for expected inflation, and other premiums for sources of risk specific to the investment, such as uncertainty about amounts and timing of future cash flows from the investment.

Interest rates are often stated as simple annual rates, even when compounding periods are shorter than one year. With *m* compounding periods per year and a stated annual rate of *i*, the effective annual rate is calculated by compounding the periodic rate (i/m) over *m* periods (the number of periods in one year).

$$\text{effective annual rate} = \left(1 + \frac{i}{m}\right)^{m} - 1$$

With a stated annual rate of 12% (0.12) and monthly compounding, the effective

$$\text{rate} = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 12.68\%.$$

Future value (FV) is the amount to which an investment grows after one or more compounding periods.

- *Compounding* is the process used to determine the future value of a current amount.
- The *periodic rate* is the nominal rate (stated in annual terms) divided by the number of compounding periods (i.e., for quarterly compounding, divide the annual rate by four).
- The *number of compounding periods* is equal to the number of years times the frequency of compounding (i.e., for quarterly compounding, multiply the number of years by four).

$$\text{future value} = \text{present value} \times (1 + \text{periodic rate})^{\text{number of compounding periods}}$$

Present value (PV) is the current value of some future cash flow.

- *Discounting* is the process used to determine the present value of some future amount.
- *Discount rate* is the periodic rate used in the discounting process.

$$\text{present value} = \frac{\text{future value}}{(1 + \text{periodic rate})^{\text{number of compounding periods}}}$$

For *non-annual compounding* problems, divide the interest rate by the number of compounding periods per year, *m*, and multiply the number of years by the number of compounding periods per year.

An *annuity* is a stream of equal cash flows that occur at equal intervals over a given period. A corporate bond combines an annuity (the equal semiannual coupon payments) with a lump sum payment (return of principal at maturity).

- *Ordinary annuity*. Cash flows occur at the end of each compounding period.
- *Annuity due*. Cash flows occur at the beginning of each period.

Present value of an ordinary annuity. Answers the question: How much would an annuity of $*X* every (month, week, quarter, year) cost today if the periodic rate is *I*%?

The present value of an annuity is just the sum of the present values of all the payments. Your calculator will do this for you.

- N = number of periods.
- I/Y = interest rate per period.
- PMT = amount of each periodic payment.
- FV = 0.
- Compute (CPT) present value (PV).

In other applications, any four of these variables can be entered in order to solve for the fifth. When both present and future values are entered, they typically must be given different signs in order to calculate N, I/Y, or PMT.

Future value of an ordinary annuity. Just change to PV = 0 and CPT → FV.

If there is a mismatch between the period of the payments and the period for the interest rate, adjust the interest rate to match. Do not add or divide payment amounts. If you have a *monthly payment*, you need a *monthly interest rate*.

Present and Future Value of an Annuity Due

When using the TI calculator in END mode, the PV of an annuity is computed as of t = 0 (one period prior to the first payment date, t = 1) and the FV of an annuity is calculated as of time = N (the date of the last payment). With the TI calculator in BGN mode, the PV of an annuity is calculated as of t = 0 (which is now the date of the first payment) and the FV of an annuity is calculated as of t = N (one period after the last payment). In BGN mode the N payments are assumed to come at the beginning of each of the N periods. An annuity that makes N payments at the beginning of each of N periods, is referred to as an annuity due.

Once you have found the PV(FV) of an ordinary annuity, you can convert the discounted (compound) value to an annuity due value by multiplying by one plus the periodic rate. This effectively discounts (compounds) the ordinary annuity value by one less (more) period.

$$PV_{\text{annuity due}} = PV_{\text{ordinary annuity}} \times (1 + \text{periodic rate})$$

$$FV_{\text{annuity due}} = FV_{\text{ordinary annuity}} \times (1 + \text{periodic rate})$$

Perpetuities are annuities with infinite lives:

$$PV_{\text{perpetuity}} = \frac{\text{periodic payment}}{\text{periodic interest rate}}$$

Preferred stock is an example of a perpetuity (equal payments indefinitely).

Present (future) values of any series of cash flows is equal to the sum of the present (future) values of each cash flow. This means you can break up cash flows any way

that is convenient, take the PV or FV of the pieces, and add them up to get the PV or FV of the whole series of cash flows.

Discounted Cash Flow Applications

Cross-Reference to CFA Institute Assigned Reading #6

Net Present Value (NPV) of an Investment Project

For a typical investment or capital project, the NPV is simply the present value of the expected future cash flows, minus the initial cost of the investment. The steps in calculating an NPV are:

- *Identify* all outflows/inflows associated with the investment.
- *Determine* discount rate appropriate for the investment.
- *Find PV* of the future cash flows. Inflows are positive and outflows are negative.
- *Compute* the sum of all the discounted future cash flows.
- *Subtract* the initial cost of the investment or capital project.

$$NPV = \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_{t-1}}{(1+r)^{t-1}} + \frac{CF_t}{(1+r)^t} - NI$$

CF_t = the expected net cash flow at time t
r = the discount rate = opportunity cost of capital
NI = the net (time = 0) investment in the project

With uneven cash flows, use the CF function.

Computing IRR

IRR is the discount rate that equates the PV of cash inflows with the PV of the cash outflows. This also makes IRR the discount rate that results in NPV equal to zero. In other words, the IRR is the *r* that, when plugged into the above NPV equation, makes the NPV equal zero.

When given a set of equal cash inflows, such as an annuity, calculate IRR by solving for I/Y.

When the cash inflows are uneven, use CF function on calculator.

Example:

Project cost is \$100, CF_1 = \$50, CF_2 = \$50, CF_3 = \$90. What is the NPV at 10%? What is the IRR of the project?

Answer:

Enter CF0 = –100, C01 = 50, F01 = 2, C02 = 90, F02 = 1.

NPV, 10, enter, ↓, CPT, display 54.395.

IRR, CPT, display 35.71 (%).

NPV vs. IRR

- *NPV decision rule*: For independent projects, adopt all projects with NPV > 0. These projects will increase the value of the firm.
- *IRR decision rule*: For independent projects, adopt all projects with IRR > required project return. These projects will also add value to the firm.

NPV and IRR rules give the same decision for independent projects.

When NPV and IRR rankings differ, rely on NPV for choosing between or among projects.

Money-Weighted vs. Time-Weighted Return Measures

Time-weighted and money-weighted return calculations are standard tools for analysis of portfolio performance.

- *Money-weighted return* is affected by cash flows into and out of an investment account. It is essentially a portfolio IRR.
- *Time-weighted return* is preferred as a manager performance measure because it is not affected by cash flows into and out of an investment account. It is calculated as the geometric mean of subperiod returns.

Various Yield Calculations

Bond-equivalent yield is two times the semiannually compounded yield. This is because U.S. bonds pay interest semiannually rather than annually.