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LEVEL 1

Book 4

Asset Valuation: Markets, Equity, Debt, Derivatives, & Alternative Investments

CONTENTS – BOOK 4
ASSET VALUATION

CONTENTS

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DEBT INVESTMENTS
STUDY SESSIONS 14 & 15

STUDY SESSION 14

STUDY SESSION 15

LEARNING OUTCOME STATEMENTS

The AIMR® Learning Outcome Statements (LOS) are listed below. These are repeated in each summary; however, the order may have been changed in order to get a better fit with the flow of the article.

STUDY SESSION 14

1.A. *Features of Fixed Income Securities* (Pages 151-170)

a. describe the basic features of a bond (e.g., maturity, coupon rate, par value, provisions for paying off bonds, currency denomination, options granted to the issuer or investor);
b. explain the purposes of a bond’s indenture and describe affirmative and negative covenants;
c. identify and describe the diversity of coupon rate structures (e.g., zero-coupon bonds, step-up notes, deferred coupon bonds, floating-rate securities);
d. describe the structure of floating-rate securities (i.e., the coupon formula and caps and floors) and the different types of floating-rate securities (e.g., inverse floaters, dual-indexed floaters, ratchet bonds, stepped spread floaters, non-interest rate index floaters);
e. define accrued interest, full price, and clean price;
f. describe the provisions for paying off bonds, and distinguish between a nonamortizing security and an amortizing security;
g. explain the provisions for early retirement of debt, including call and refunding provisions, prepayment options, sinking fund provisions, and the construct of index amortizing notes;
h. distinguish between a nonrefundable and a noncallable bond;
i. explain the difference between a regular redemption price and a special redemption price;
j. identify embedded options (e.g., call provision, prepayment provision, accelerated sinking fund provision, put option, conversion option) and explain whether such options benefit the issuer or the bondholder;
k. explain the importance of options embedded in a bond issue;

FORMULA SHEET – DEBT INVESTMENTS STUDY SESSIONS 14 & 15

FABOZZI, CHAPTER 1

annual (coupon) interest = $\frac{c}{100}$ x F

floating rate coupon formula = reference rate +/– quoted margin

deleveraged floater coupon rate = *b* × reference rate + quoted margin

inverse floater coupon rate = constant rate (K) – L × reference rate

dual indexed floater coupon rate = reference rate_1 – reference rate_2 + fixed percentage margin

FABOZZI, CHAPTER 2

premium bond: coupon rate > required market yield ⇒ bond price > par value

discount bond: coupon rate < required market yield ⇒ bond price < par value

par bond: coupon rate = required market yield ⇒ bond price = par value

callable bond value = value of the straight bond component – value of the embedded call option

approximate *percentage* change in price (for 100 basis point change in rates)

$$= \frac{(\text{price when yields fall} - \text{price when yields rise})}{2 \times (\text{initial price}) \times (\text{yield change in decimal})}$$

approximate *dollar* change in price = (effective duration × current bond (portfolio) value) / 100

FABOZZI, CHAPTER 3

TIPS coupon: *coupon payment* = adjusted principal at the *beginning* of the period × (1 + *i*) × *c*
= adjusted principal at the *end* of period × *c*

FABOZZI, CHAPTER 4

yield interpolation formula:

$\text{yield}_n = \text{yield}_{\text{nearest lower maturity}} + (\text{Yield}_{\text{nearest higher maturity}} - \text{Yield}_{\text{nearest lower maturity}}) \times$
(n – nearest lower maturity)/(nearest higher maturity – nearest lower maturity)

absolute yield spread = yield on bond 1 of maturity “t” – yield on bond 2 of maturity ‘t’

relative yield spread = absolute yield spread / yield on bond 2

yield ratio = yield on bond 1 / yield on bond 2 = relative yield spread + 1

INTRODUCTION TO THE MEASUREMENT OF INTEREST RATE RISK

Study Session 15
Fabozzi, Chapter 7

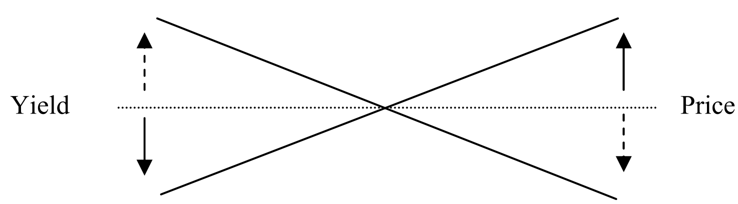
EXECUTIVE SUMMARY

This chapter deals with *bond price volatility* and ways to measure it. The discussion begins with a brief look at the typical *price/yield relationship* that exists, first on noncallable bonds (which possess *positive convexity)* and then on callable bonds (which exhibit *negative convexity).* The rest of the chapter—indeed, the vast majority of it—addresses *ways to assess/measure bond price volatility*. And in this regard, probably no two issues are more important than *Duration* and *Convexity!* In addition to knowing how to compute and apply duration and convexity, it's also important to have a thorough understanding of the properties and determinants of these two widely used measures of volatility.

A. Introduction

Debt securities that do not contain embedded options, i.e., they are not callable, putable, convertible, etc.—will exhibit the following price/yield relationship:

Figure 1



That is, price and yield will move in the opposite direction. However, the degree of the price change is not always the same. Given that changes in required yields will cause bond prices to change, but that the degree of change is not the same for all bonds, it is desirable to develop a quantitative measure of *price volatility or interest rate risk.*

This chapter explores three approaches to measuring the price volatility of a bond as caused by movements in interest rates.

- The Full Valuation Approach.
- The Duration/Convexity Approach.
- The Price Value of a Basis Point Approach.

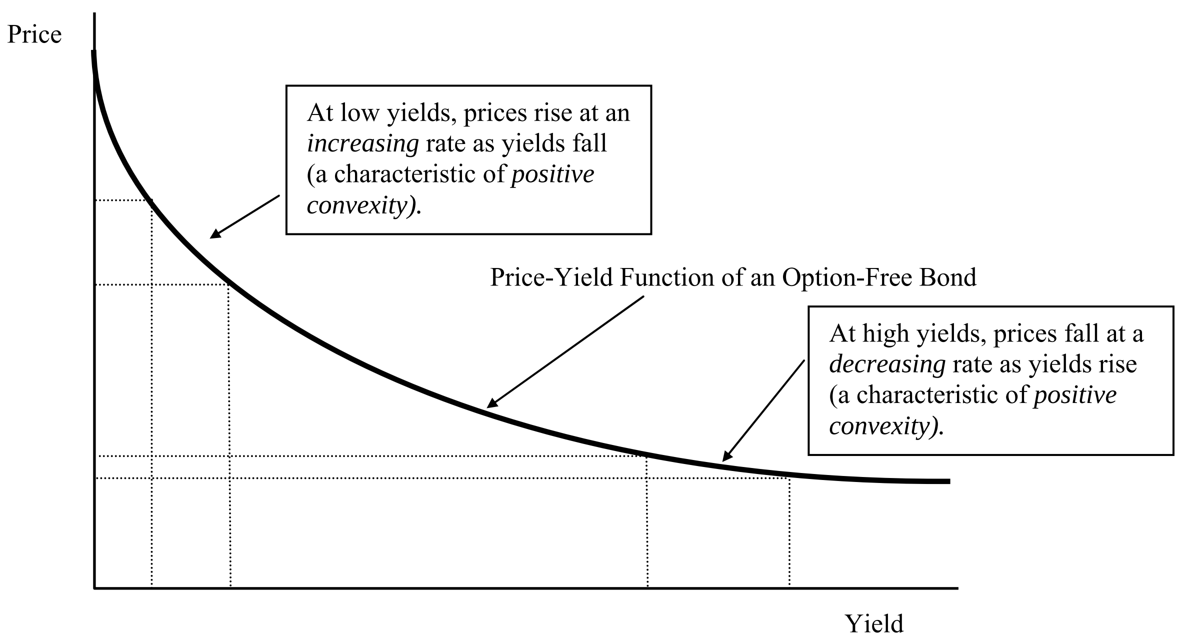
B. Price-Volatility Characteristics of Bonds

LOS 1.C.c: Explain and illustrate the price volatility characteristics for option-free bonds when interest rates change (including the concept of "positive convexity").

1. The Price/Yield Relationship for Bonds

As noted above, the price of a bond is simply the present value of its future cash flow stream, discounted at a given required rate of return (or yield). When this yield changes from one level to another, the bond's future cash flows are not affected, but the present value of those cash flows are calculated using the new discount rate (yield). Thus, if the discount rate goes up, the present value of the future cash flows (price) goes down; this explains why *there is a negative correlation between bond price and yield.* This behavior is known as the *price/yield relationship* and it's typical of all bonds—that is, *price and yield move in opposite directions.* The graph below illustrates the price/yield relationship of a typical *option-free (noncallable) bond,* and shows graphically how the price of a bond responds to changes in yield. As can be seen, this relationship is not linear, but *convex.* In fact, *one of the key properties embedded in noncallable bonds is that they all exhibit positive convexity,* meaning that prices go up faster than they go down!

Figure 2



In addition to the inverse relationship between bond prices and yields, the following are some other properties that impact *bond price volatility*.

- Although the *direction of change in price* for a given change in yield is the same for all bonds, *the magnitude of the price change differs across bonds.*

- For *very small changes in yield* (of less than 50 basis points or so), the magnitude of the percentage price change for different bonds is *about equal, whether the yield increases or decreases.*
- For *large changes in yield* (of more than 50 basis points or so), the magnitude of the percentage price change *depends on whether the yield increases or decreases.* Note that the percentage price *decrease* associated with a given increase in the yield is *less* than the percentage price *increase* associated with an equal decrease in the yield; this, of course, is a characteristic of *positive convexity.*

There's one other point you should be aware of with regard to bond price volatility. The magnitude of the *percentage price change on a bond for a given change in interest rates* depends on three features: (1) the bond's coupon rate; (2) its term to maturity; and (3) the initial yield. In particular:

- *The lower the coupon, the greater the bond price volatility.*
- *The longer the term to maturity, the greater the price volatility.*
- *The lower the initial yield, the greater the price volatility.*

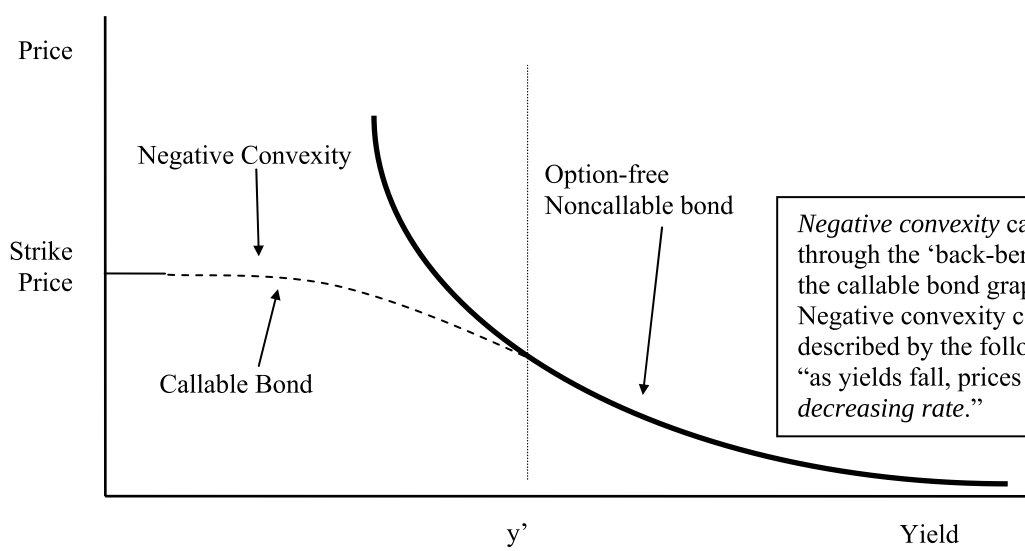
2. Callable Bonds

Fixed-income securities are often issued with *embedded options, such as call features.* When this happens, the price/yield relationship will change and so will the price volatility characteristics of the issue.

A call option gives *the issuer* the right to buy back the bond at a fixed price(s) at some point(s) in the future, prior to the date of maturity. The investor takes a short position in the call, so the right to purchase rests with the issuer; such bonds are deemed to be *callable*.

LOS 1.C.d: Explain and illustrate the price volatility characteristics of callable bonds and prepayable securities when interest rates change (including the concept of "negative convexity").

Figure 3



Negative convexity can be illustrated through the 'back-bending' portion of the callable bond graph to the left. Negative convexity can also be described by the following statement: "as yields fall, prices rise at a *decreasing rate*."

For an option-free (noncallable) bond, prices will fall as yields rise, and more important perhaps, *prices will rise unabated as yields fall*—in other words, they'll move in line with yields. That's not the case, however, with *callable bonds.* For as can be seen in Figure 3, with callable bonds, the decline in yield will reach the point where *the rate of increase in the price of the bond will start slowing down and eventually level off;* this is known as *negative convexity.* Such behavior is due to the fact that the issuer has the right to retire the bond prior to maturity at some specified call price. That call price, in effect, acts to hold down the price of the bond (as rates fall) and causes the price/yield curve to flatten! As can be seen, the point where the curve starts to flatten is at (or near) a yield level of y'.

Below yield level y', investors begin to anticipate that the firm may call the bond. If the firm calls the bond, investors will receive the call price. Therefore, as yield levels drop, the bond's market value is bounded from above by the call price. Thus, callability effectively caps the investor's capital gains as yields fall. Moreover, it exacerbates reinvestment risk, since it increases the cash flow which must be reinvested at lower rates (i.e., without callablility, the CF will only be the coupon; with callability, the CF is the coupon plus the call price). Thus, in Figure 3, so long as yields remain *below level y',* callable bonds will exhibit price compression, or *negative convexity;* however, at yields *above level y',* those same callable bonds will exhibit all the properties of *positive convexity!*

C. The Full Valuation Approach

LOS 1.C.a: Distinguish between the full valuation approach and the duration/convexity approach for measuring interest rate risk, and explain the advantage of using the full valuation approach.

The *most straightforward method* for measuring interest rate risk is the so-called full valuation approach. Essentially this boils down to the following steps:

- Begin with the current market yield and price.
- Estimate hypothetical changes in required yields.
- Recompute bond prices using the new required yields.
- Compare the resulting price changes.

Note that all the analyst is doing here is computing a series of hypothetical bond prices, given a series of market yields that are expected to exist in the near future. Thus, the analyst is simply trying to find out what would happen to the price of a bond if yields rose (or fell) to this level (or that). It's just another basic application of what you learned in Chapter 5 (pricing bonds).

LOS 1.C.b: Compute the interest rate risk exposure of a bond position or of a bond portfolio, given a change in interest rates.

This approach is illustrated in Figure 4, first for Bond X, second for Bond Y, and third for a two-bond portfolio comprising positions in X and Y.

Consider two option-free bonds: X is an 8 percent annual-pay bond with 5 years to maturity, priced at 108.4247 to yield 6 percent. (N = 5; PMT = 8.00; FV = 100; I/Y = 6.00%; CPT → PV → –108.4247). Y is a 5 percent annual-pay bond with 15 years to maturity, priced at 81.7842 to yield 7 percent. You have a $10 million face-value position in each, and are evaluating two scenarios. The first is a parallel shift in the yield curve of +50 basis points, and the second is a parallel shift of +100 basis points. Note that the bond price of 108.4247 is the price per $100 of par value. Hence, with $10 million of par value bonds, the market value will be $10.84247 million.

Figure 4

		Market Value of			*Portfolio*
Scenario	*Yield Δ*	*Bond X (in millions)*	*Bond Y (in millions)*	*Portfolio*	*Value Δ%*
Current	+0 bp	$10.84247	$8.17842	$19.02089	–0.00%
1	+50 bp	$10.62335	$7.79322	$18.41657	–3.18%
2	+100 bp	$10.41002	$7.43216	$17.84218	–6.20%

N = 5; PMT = 8; FV = 100; I/Y = 6% + 0.5% → CPT PV = 106.2335
N = 5; PMT = 8; FV = 100; I/Y = 6% + 1% → CPT PV = 104.1002
N = 15; PMT = 5; FV = 100; I/Y = 7% + 0.5% → CPT PV = 77.9322
N = 15; PMT = 5; FV = 100; I/Y = 7% + 1% → CPT PV = 74.3216
Portfolio value change 50 bp: (18.41657 – 19.02089 / 19.02089 = –0.03177
Portfolio value change 100 bp: (17.84218 – 19.02089 / 19.02089 = –0.06197

Also, on an individual bond basis, it's worth noting that Bond X does far less damage to the portfolio than Bond Y—i.e., with a 50bp increase in yields, X falls by 2.02 percent while Y falls by 4.71 percent; and with a 100bp increase, X falls by 3.99 percent while Y drops by 9.12 percent. This, of course, is totally predictable since Bond Y is a much *longer bond,* and it has a *lower coupon*—all of which means *more price volatility!*

D. Duration

LOS 1.C.i: Describe the various ways that duration has been interpreted and why duration is best interpreted as a measure of a bond's or portfolio's sensitivity to changes in interest rates.

Duration is, by far, the *most widely used* measure of bond price volatility! Basically, it shows how the price of a bond is likely to react to different interest rate environments. As noted earlier, a bond's price volatility is a function of its coupon, maturity, and initial yield. Well, *duration captures the impact of all three of these variables in a single measure.* Just as important, *a bond's duration and its price volatility are directly related*—i.e., the longer the duration, the more price volatility there is in a bond. Such a characteristic, of course, greatly facilitates the comparative evaluation of potentially competitive bond investments.

Duration is sometimes described as the first derivative of the bond's price function with respect to yield (i.e., how the measure is derived mathematically), or as a present value-weighted number of years to maturity. Neither of these descriptions is particularly useful in explaining how duration is used in practice. The most concise, useful description is that *duration is a measure of a bond's (or portfolio's) sensitivity to a 1 percent change in interest rates.* One way to calculate duration is as follows:

$$\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)}$$

where:
V_- = estimated price if yield decreases by a given amount, Δy
V_+ = estimated price if yield increases by a given amount, Δy
V_0 = initial observed bond price
Δy = change in required yield, in decimal form

The above equation provides a measure which allows us to approximate the percentage change in the price of a bond for a 100 basis point (1.00 percent) change in required yield, assuming that the shift in the yield curve is parallel. *(Editor's Note: This is the first of four duration/convexity equations that you need to know for this material!)*

LOS 1.C.g: Distinguish among modified duration, effective (or option-adjusted) duration, and Macaulay duration. (*Editor's Note: The duration computation remains the same. The only difference between modified and effective duration is that effective duration is used for bonds with embedded options.)*

The duration formula shown above is often referred to as *effective duration.* It's widely used because it can be used with *either noncallable* or *callable bonds;* or for that matter, on bonds with any other type of embedded option. Another way to measure duration is to use what's known as *modified duration.* This measure is sometimes found by first computing a bond's "Macaulay" duration, and then adjusting it for the bond's YTM. *(Editor's Note: For Level I, you do not need to know the equations for Macaulay/modified duration, so as you review the material on page 263 of Fabozzi, don't get involved in all the arithmetic.)* The fact is, for noncallable bonds, it doesn't make any difference which measure (effective or modified) you use; they both yield the same duration value! That's why, in Chapter 7 of the Fabozzi text, the terms "effective" and "modified" are used interchangeably when describing duration on noncallable bonds. With callable bonds, however, (or bonds with other types of embedded options) it does make a difference which measure you use—modified duration simply doesn't work with such bonds. That's why attention here is centered on effective duration: it can be used with both noncallable and callable bonds. *(Even so, note that at Level I, almost without exception, duration is measured for noncallable bonds.)*

LOS 1.C.e: Compute the duration of a bond, given information about how the bond's price will increase and decrease for a given change in interest rates.

Example: Computing Effective/Modified Duration

Suppose that there is a 15-year option free (noncallable) bond with an annual coupon of 7 percent trading at par. If interest rates rise by 50 basis points (0.50%), the estimated price of the bond will be 95.586 percent. (N = 15; PMT = 7.00; FV = 100; I/Y = 7.50%; CPT → PV → –95.586). What's the bond's effective duration?

Answer:

Given the information above, plus the fact that if interest rates fall by 50 basis points, the estimated price of the bond will be 104.701, we can calculate the bond's duration as:

$$\text{duration} = \frac{104.701 - 95.586}{2(100)(0.005)} = 9.115$$

What this tells us is that for a 100 basis point, or a 1.00 percent change in required yield, the expected price change is 9.115 percent. In other words, the price of this bond should go up or down by approximately 9.115 percent for every 1 percent change (decrease or increase) in the bond's required yield—i.e., if the yield-to-maturity (YTM) on this bond goes up by 1 percent, the price should fall by around 9.115 percent and if YTM drops by 1 percent, the price of the bond should rise by 9.115 percent.

1. Using Effective/Modified Duration to Estimate a Price Change

LOS 1.C.f: Compute the approximate percentage price change for a bond, given the bond's duration and a specified change in yield.

Once you've computed a bond's duration (per the above equation), you can use that information *to see how the price of the bond will respond to a specific change in yield.* This is sometimes known as the *duration effect,* as it shows the percentage change in price due to duration; it can be found as follows:

approximate *percentage* change in price due to duration = (–)(duration)(Δy)

Note, the negative sign in the equation—it's there because bond prices and yields move in opposite directions. *(Editor's Note: This is the 2nd duration/convexity equation that you need to know for this material!)*

Example: *Estimating Price Changes With Effective/Modified Duration*

Suppose that the 15-year, 7 percent, option-free bond *from the previous example* is currently trading at par and we want to estimate the price change if yields fall or rise by 150 basis points.

Using the previously computed value for effective duration, we get:

percentage Δ in price due to duration (given a 150bp *DROP* in yield) = –9.115 × (–1.50%) = 13.6725% (results in an estimated price of 113.6725)

actual price change = 15.0564% (actual price of 115.0564) percentage Δ in price due to duration (given a 150bp *INCREASE* in yield) = –9.115 × (1.50%) = –13.6725% (results in an estimated price of 86.3275)

actual price change = –12.4564% (actual price of 87.5436)

Estimated price computations:

with $\Delta y = -1.50\%$: $100.000 + 13.6725 = 113.6725$
with $\Delta y = +1.50\%$: $100.000 - 13.6725 = 86.3275$

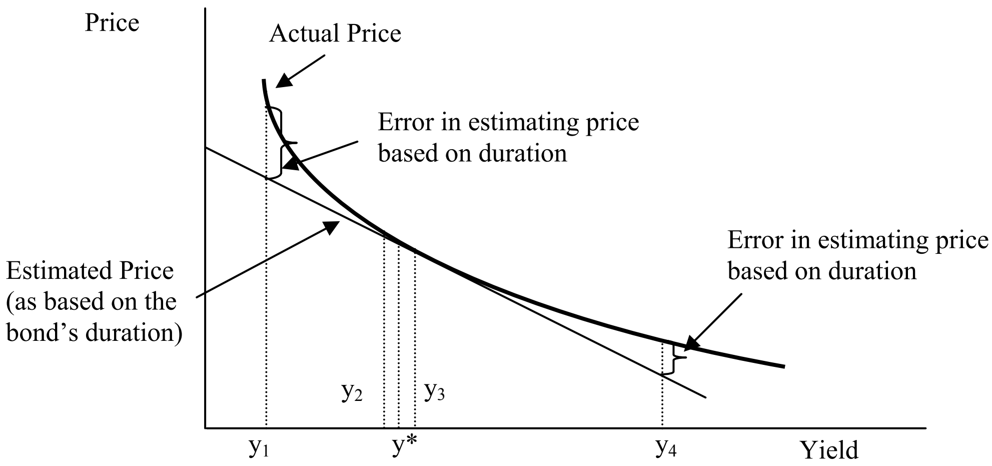
Actual price computations:

with Δy = +1.50%: N = 15; PMT = 7; FV = 100; I/Y = 7% + 1.5% → CPT PV = 87.5436
with Δy = –1.50%: N = 15; PMT = 7; FV = 100; I/Y = 7% – 1.5% → CPT PV = 115.0564

Note in the above calculations that the estimated price changes differ from the actual price changes, regardless of whether yields go up or down. For *large changes in yield,* that's a very common behavioral characteristic of duration—that is, for wide swings in yield (of 50-100 basis points or more), duration tends to *underestimate the increase in price* that occurs with a decrease in yield, and *overestimate the decrease in price* that comes with an increase in yield (to confirm this, just look at the above example). That's not the case, however, with very small changes in yield (of 10-25 basis points, or so); under those conditions, the estimated and actual price changes equal (or are very close to) one another.

We can see why the estimates vary from actual prices by referring to the price/yield graph in Figure 5. Note in the graph that the top (curved) line represents the actual price behavior of a noncallable bond (as you'd expect, it's convex in shape) and that it lies above the straight line, which represents the estimated price behavior of the bond using the effective/modified measure of duration. Observe that as the market yield moves from y* to y_2 or y_3, there is *virtually no difference* between the curved line and the straight line—implying that the actual and estimated price changes are pretty much the same. (As proof, repeat the example above for a 10 basis point change in yield, and you'll find an estimated price of 99.12 vs. an actual price of 99.10 when rates rise, and an estimated price of 100.88 vs. an actual of 100.92 when rates fall.) For larger swings in yield—e.g., when yields move from y* to either y_1 or y_4—the differences (or magnitude of error) between estimated and actual prices become quite substantial.

Figure 5



The error in the estimate is due to the curvature of the actual price path. *The larger the change in yield, the larger the error*. This is due to the degree of convexity. If we can generate a measure of this convexity, we can use this to improve our estimate of bond price changes. This is where convexity comes in, a measure of bond price volatility that we will explore in the next section.

2. Effective Duration for Callable Bonds

LOS 1.C.h: Explain why effective duration, rather than modified duration or Macaulay duration, should be used to measure the interest rate risk for bonds with embedded options.

Modified duration assumes that the cash flows on the bond will not change (i.e., that we're dealing with a noncallable bond). This differs from *effective duration*, which considers expected *changes in cash flows* that may occur for bonds with (or without) embedded options.

We can use the same duration formula as above to calculate effective duration of, a callable bond. The only difference is that V_- and/or V_+ may be affected by changes in cash flows that result from the presence of an embedded option (e.g., a call feature).

Example: Effective Duration Calculation

We will use the same data in our previous duration calculation, except that we will now assume that the bond is callable at 102.50. That is, the bond now has an embedded option feature. We will also assume that its price cannot exceed the call price.

Answer:

Therefore, V_- will have a value of 102.50, as opposed to the value of 104.701, which was used to calculate modified duration.

$$Duration = \frac{102.50 - 95.586}{2(100)(0.005)} = 6.914$$

The effective duration is 6.914, compared with a modified duration of 9.115 (as per our earlier calculation). Note that the difference in duration is due to differences in the price path as interest rates fall. That is, in the first example, the noncallable bond had *positive convexity,* while in this case, the callable bond has *negative convexity.*

E. Convexity

Modified (or effective) duration is a good approximation of potential bond price behavior, but it's only good for relatively small changes in interest rates. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more prevalent, meaning that a linear estimate of price changes will contain errors. Modified duration is a linear estimate, as it assumes that the price change will be the same regardless of whether interest rates go up or down. Take another look at the price/yield graph, as shown in Figure 5. We know that the relationship between bond price and yield is not linear (as assumed by duration), but rather convex—which, of course, explains why the differences in the actual and estimated price lines widens as the yield swings grow. That is, the widening error in the estimated price is due to the *curvature of the actual price path;* this is known as the *degree of convexity.* Fortunately, the amount of convexity in a bond can be measured and *used to supplement duration* in order to achieve a more accurate estimate of the change in price. *It's important to note that all that convexity does is account for the amount of error in the estimated price* (as based on duration). In other words, it picks up where duration leaves off and basically converts the straight (estimated price) line into a curved line that more closely resembles the convex (actual price) line.

LOS 1.C.j: Compute the convexity measure of a bond, given information about how the price will increase and decrease for a given change in interest rates.

While a precise calculation of a convexity involves the use of calculus (convexity is the second derivative of the price function with respect to yield), we can generate an approximate measure of convexity as follows:

$$\text{convexity} = \frac{V_- + V_+ - 2V_0}{2V_0\,(\Delta y)^2}$$

where:
V_- = estimated price if yield decreases by a given amount, Δy
V_+ = estimated price if yield increases by a given amount, Δy
V_0 = initial observed bond price
Δy = change in required yield, in decimal form

Given the magnitude of the change in yield, the above equation essentially helps us define the amount of convexity in the price/yield relationship. *(Editor's note: This is the 3rd duration/convexity equation that you need to know for this material!)*

Example: Computing Convexity

Suppose there is a 15-year option free (noncallable) bond with an annual coupon of 7 percent trading at par. If interest rates rise by 50 basis points (0.50%), the estimated price of the bond is 95.586 percent. If interest rates fall by 50 basis points, the estimated price of the bond is 104.701 percent.

Answer:

Given the above information, we can calculate the convexity of this bond as follows:

$$\text{convexity} = \frac{104.701 + 95.586 - 2(100)}{2(100)(0.005)^2} = 57.4$$

Unlike duration, this value (57.4 in the example above) cannot conveniently be converted into some measure of potential price volatility. Indeed, the convexity value in the above example means nothing in isolation—other than *a higher number means more price volatility than a lower number.* This value can become very useful, however, when it's used to measure a bond's *convexity effect*—a measure that can then be combined with a bond's duration to provide a more accurate estimate of potential price change.

LOS 1.C.k: Estimate a bond's percentage price change, given the bond's duration and convexity and a specified change in interest rates.

Using Convexity to Improve Price Change Estimates

Estimating the Convexity Effect. The calculated convexity for our bond, as per the above equation, is 57.4. (Note that given the information in the problem—especially the starting price of this noncallable bond, V_0, and the implied market yield that accompanies it–you'd end up with the same measure of convexity (57.4) regardless of what number you use for the change in yield (Δy)—it'd be the same whether you used 25, 50, 100, or 150 basis points). So, to obtain an estimate of *the percentage change in price due to convexity*—that is, the amount of price change that's not explained by duration—we make the following calculation (recall that in our previous example, interest rates were going to either rise or fall by 150bp):

convexity effect (or, the approximate percentage change in price due to convexity)
= convexity × $(\Delta y)^2$
= 57.4 × $(0.015)^2$ = 0.012915 or 1.2915%

Note: you must always use the decimal representation of the change in interest rates when computing the convexity adjustment.

(Editor's Note: This is the 4th duration/convexity equation that you need to know from this material!)

As can be seen here, *the convexity effect is pretty small*—only 1.29 percent in this case. That's a common trait of convexity; but remember, this measure is simply correcting for the error embedded in duration, so you'd expect convexity to be a much smaller measure than duration. Also note that for an option-free (noncallable) bond, the convexity effect is *always positive,* no matter which direction interest rates move; though it can be negative if the bond has embedded options. Thus, for noncallable bonds, convexity is always added to duration to modify (i.e., correct) the price volatility errors embedded in duration. By always adding convexity to duration, it decreases the drop in price (due to an increase in yields) and adds to the rise in price (due to a fall in yields). *This is a very important property of convexity,* and gives another meaning to the term "positive convexity."

Now, by combining duration and convexity, we can obtain a far more accurate estimate of the percentage change in price of a bond, especially for large swings in yield. That is, you can account for the amount of convexity embedded in a bond by *adding the convexity effect to duration,* as follows:

$$\text{percentage change in price} = \text{duration effect} + \text{convexity effect} = \left[-\text{duration} \times (\Delta y)\right] \text{plus} \left[\text{convexity} \times (\Delta y)^2\right]$$

Example: Estimating Price Changes With Both Duration and Convexity

By combining the two measures, we get the following:

$$\text{Est.}\left[\Delta V_{-}\%\right] = \left[-9.115 \times (-1.50\%)\right] + \left[57.4 \times (0.015)^2\right] = 13.6725 + 1.2915 = 14.964\%$$

$$\text{Est.}\left[\Delta V_{+}\%\right] = \left[-9.115 \times (1.50\%)\right] + \left[57.4 \times (0.015)^2\right] = -13.6725 + 1.2915 = -12.381\%$$

Given the percentage price changes computed above, we would have an estimated price of 114.964 (vs. an actual of 115.056) for a 150bp drop in yield, and an estimated price of 87.619 (vs. an actual of 87.544) for a 150bp increase in yield. Clearly, *the estimates are a lot more accurate than using duration alone.* But there's one other thing that's happened—i.e., the price behavior of the bond is now exhibiting properties of *positive convexity.* For as can be seen in the example above, the price of the bond is now expected to go up (+14.964%) faster than it's expected to go down (–12.381%)!

LOS 1.C.l: Differentiate between modified convexity and effective convexity. (*Editor's Note: There is no such thing as "modified" convexity. The only reason that we have "modified" duration is that the original Macaulay's duration had a flaw in it and needed to be "modified" by dividing it by [1 + yield / 2]).*

Example: Effective Convexity of a Callable Bond

We can also calculate *effective convexity*, using the same methodology as above. This requires an adjustment in the estimated bond values (V_-, V_+) to reflect any *change in estimated cash flows,* due to the presence of embedded options, as was done to calculate effective duration. *If we assume that the bond is callable,* and that the maximum value for V_- is 102.50 percent, we can calculate the effective convexity of the bond to be:

$$\text{effective convexity} = \frac{102.50 + 95.586 - 2(100)}{2(100)(0.005)^2} = -382.8$$

This figure is negative because the price is increasing at a decreasing rate as yields fall below the coupon rate for this callable bond.

F. The Price Value of a Basis Point

LOS 1.C.m: Compute the price value of a basis point (PVBP) and explain its relationship to duration.

Also known as the *dollar value of an 01,* or DV01, the *price value of a basis point* (PVBP) is a measure of bond price volatility that shows *the extent to which the price of a bond will change when the required yield changes by one basis point (bp).* PVBP is an absolute value that's measured as follows:

PVBP = initial price – price if yield is changed by 1 bp

Example:

Consider a 5 year, 9 percent semiannual pay bond with a par value of $1,000. Calculate the PVBP for this bond.

Answer:

The bond's price with a 9% yield is obviously par (since the 9% yield = the 9% coupon). At a yield of 9.01% (a one bp increase), the price of the bond drops to $999.6045 (i.e., N = 10; PMT = 45; FV = 1,000; I/Y = 4.505; CPT → PV = –999.6045). Thus, the PVBP is:

$1,000 – $999.6045 = **$0.3955**

Therefore, if the yield on this bond goes up, or down, by 1 bp, you can expect the price of the bond to go down, or up, by roughly 40 cents. Clearly, the higher a bond's PVBP, the higher the potential price volatility of the issue—a property that's exactly the same as duration. In fact, *PVBP is nothing more than a variation of dollar duration.* The dollar duration of a 1bp change in yield is, not surprisingly, virtually the same as the value you'd compute using PVBP.

CONCEPTUAL OVERVIEW

1. The price of an option-free bond moves inversely with changes in required yield.
2. The degree of the price change is not always the same, even for identical changes in required yield.
3. The price/yield relationship is not linear, but is curved. This is known as convexity.
4. *Callable bonds* exhibit negative convexity at yield levels where investors begin to anticipate that the firm will call the bond.
5. The *full valuation approach* is the most straightforward method of estimating interest rate risk.
6. Since the full valuation approach requires valuing each bond for each interest rate scenario, it can become unwieldy when a portfolio contains a large number of positions.
7. *Duration* is the most commonly used method of estimating interest rate risk (i.e., price volatility).
8. Duration (modified or effective) is a measure of a bond's sensitivity to a 1 percent change in interest rates.
9. Duration (modified or effective) allows one to estimate the percentage change in a bond's price for a small change in required yield.
10. Duration (modified or effective) does not provide good estimates of price change when there are large changes in required yield. This is because of the convexity in the price/yield relationship.
11. *Effective duration* is the same as modified duration, except that it considers expected changes in cash flows that may occur for bonds with embedded options.
12. *Convexity* is a measure of the degree of curvature or convexity in the price/yield relationship.
13. Combining the standard convexity measure with a measure of duration provides more accurate estimates of bond price changes, particularly when the change in required yields is relatively large.
14. Effective convexity is the same as standard convexity, except that it considers expected changes in cash flows that may occur for bonds with embedded options.
15. *PVBP* shows what happens to the price of a bond given a 1 basis point change in yield, and is another way of measuring a bond's potential price volatility. PVBP = initial price – price if yield is changed by 1 bp.

PROBLEM SET: INTRODUCTION TO THE MEASUREMENT OF INTEREST RATE RISK

1. Why is the price/yield profile of a callable bond less convex than that of an otherwise identical option-free bond? The price:
   A. increase is capped from above at or near the strike price as the required yield decreases.
   B. increase is capped from above at or near the strike price as the required yield increases.
   C. decrease is limited from below at or near the strike price as the required yield decreases.
   D. decrease is limited from below at or near the strike price as the required yield increases.

2. You own $15 million face of the 4.65 percent semiannual-pay Portage Health Authority bonds. The bonds have exactly 17 years to maturity and are currently priced to yield 4.39 percent. Use the full valuation approach to compute the interest rate exposure (in percent of value) for this bond position given a 75 basis point increase in required yield.
   A. –9.104%.
   B. –9.031%.
   C. –8.344%.
   D. –8.283%.

3. You are estimating the interest rate risk of a 14 percent semiannual-pay coupon with 6 years to maturity. The bond is currently trading at par. Use a 25 basis point change in yield to compute the modified duration of the bond.
   A. 0.389.
   B. 0.397.
   C. 3.889.
   D. 3.970.

4. Suppose that the bond in question 3 is callable at par today. Using a 25 basis point change in yield, compute the bond's effective duration assuming that its price cannot exceed 100.
   A. 1.972.
   B. 1.998.
   C. 19.72.
   D. 19.98.

5. Referring to the bond in Question 3. Suppose that the bond is option-free. Compute its convexity value. (Again, use a 25bp change in yield.)
   A. 1.02.
   B. 10.2.
   C. 10.4.
   D. 100.4.

6. Suppose that you determine that the modified duration of a bond is 7.87. Estimate the percentage change in price due to duration given yields decrease by 110 basis points.
   A. –8.657%.
   B. –7.155%.
   C. +7.155%.
   D. +8.657%.

7. Suppose that you've found that the convexity of a bond to be 57.3. Estimate the convexity effect if yields decrease by 110 basis points.
   A. –1.673%.
   B. –0.693%.
   C. +0.693%.
   D. +1.673%.

8. Assume you're looking at a bond that has an effective duration of 10.5 and a convexity of 97.3. Using both of these measures, find the estimated percentage change in price for this bond, given market yields are expected to decline by 200 basis points.
   A. 22.95%.
   B. 19.05%.
   C. 17.11%.
   D. 24.89%.

9. An analyst has determined that if market yields rise by 100 basis points, a certain high-grade corporate bond will have a convexity effect of 1.75 percent; further, she's found that the total estimated percentage change in price for this bond should be –13.35 percent. Given this information, it follows that the bond's percentage change in price due to duration is:
   A. –15.10%.
   B. –11.60%.
   C. +15.10%.
   D. +16.85%.

10. The total price volatility of a typical noncallable bond can be found by:
    A. adding the bond's convexity effect to its effective duration.
    B. adding the bond's negative convexity to its modified duration.
    C. subtracting the bond's negative convexity from its positive convexity.
    D. subtracting the bond's modified duration from its effective duration, then add any positive convexity.

11. An investor paid $1,029.23 for a $1,000 face value, seven year, 5.5 percent semiannual coupon bond. The bond's PVBP is *closest* to:
    A. $5.93.
    B. $0.60.
    C. $0.05.
    D. $5.74.

PROBLEM SET ANSWERS STUDY SESSIONS 14 &15

FABOZZI, CHAPTER 1 – FEATURES OF FIXED INCOME SECURITIES

1. **A** The dollar amount of the coupon payment is computed as follows:

   coupon in \$ = \$5,000 × 0.0425 (½ of 8.5%) = \$212.50

2. **B** If the quoted price is 98 5/32, this means that the dollar amount is:
   0.981563 × \$50,000 = \$49,078.15

3. **C** If the bond provides investors with a higher coupon rate than the market interest rate, the bond has to be trading at a premium relative to its par value.

4. **B** The value of the bond is computed as follows:

   bond value = \$1,000 / (1.0425)6 = \$779.01
   N = 6; I/Y = 4.25; FV = 1000; PMT = 0 → CPT PV 779.01
   The general market rule for pricing zero-coupon bonds is to treat them as if they had semiannual coupon payments.

5. **B** This value is computed as follows:

   semiannual coupon = (LIBOR + 125 basis points) / 2 = (6.5 + 1.25) / 2 = 3.875%

6. **B** A cap limits the upside potential of the coupon rate paid on the floating-rate bond and is therefore a disadvantage to the bondholder. A floor limits the downside potential of the coupon rate and is therefore a disadvantage to the bond issuer.

7. **C** The bondholder always receives coupon payments made by the issuer and not the opposite since that would imply a negative interest rate.

8. **A** Since the treasury reference rate is below the lower limit of the range for Year 2, the coupon rate is set to zero.

9. **C** The clean price is the bond price without the accrued interest, so it is equal to the quoted price 105 7/32 = \$1,052.19.

10. **B** A call provision gives the bond issuer the right to call the bond at a prespecified price. A bond issuer may want to call a bond if he is paying a high coupon and interest rates have decreased so that he would be able to get cheaper financing.

11. **B** Whenever the price of the bond increases above the strike price stipulated on the call option, it will be optimal for the issuer to call the bond. So in the limit, the price of a freely callable bond can never rise above its call strike price.

12. **B** *Principal* payments are linked to a reference rate. The other statements are true. Similar to sinking funds and call provisions, the index amortizing bond results in the payment of principal prior to maturity.

13. **B** A deferred call provision means that when issued, the issuer may not call the bond for a number of years. These bonds may be called for purposes such as the sinking fund. The other statements are false. Since Gould does not know the market rate at the time of issuance, we cannot say that the bond was issued at a premium. He would be able to conclude that the bond would trade at a premium to a similar noncallable bond. The company may not *refund* the bonds with a source of funds costing less than 6.50 percent. A refunding occurs when the issuer sells the bonds to *redeem an earlier series* of bonds. The call option benefits the issuer, not the investor. Thus, the issuer must pay a premium to investors.

14. **D** Since the bond has a nonrefunding provision (prior to January 1, 2004, the company may not redeem the bonds with a source of funds costing less than 6.50 percent), the company cannot call the bonds to lower the rate. The other statements are false. The issuer may call in certain bonds to satisfy the sinking fund provision, and the special redemption price is usually par. So, the investor is not guaranteed a premium. The cash flow stream for these bonds is unpredictable because of the call and sinking fund provisions. Valuing a complex bond such as this requires modeling interest rates and issuer and borrower behavior. Although the existence of a sinking fund does lower default risk, some bonds may be redeemed at par (special redemption price) for this purpose. The investors who have their bonds redeemed would have reinvestment risk and would receive less than market value if the bonds are trading at a premium at the time of the redemption.

FABOZZI, CHAPTER 2 – RISKS ASSOCIATED WITH INVESTING IN BONDS

1. **A** Because the 6 percent coupon on this bond exceeds the current (5 percent) market yield, this issue will be trading at a price that's above its par value (the bond will be trading at a premium). By using a hand-held calculator, the current price of the bond (i.e., where a 6 percent bond is being priced to yield 5 percent) can be

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